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matrix of blurring operator and a inverse matrix of gradient operator (integration) to the digital blurred (degraded) image according to formula (43).

An inverse sharpening filter, according to the present invention, is then numerically constructed, as represented in block 12 (FIG. 1), by use of a function  $w_e(m_{g,1})$  accomplishing a minimum gradient support constraint according to formula (45).

In partially sharpened image block 13 of image enhancement and sharpening method 10, the subsequent iteration of the weighted restored image is constructed according to formulae (47), (48), and (49).

In inverse filtering block 14, the real parameters of the ideal image are recalculated from the weighted parameters by inverse filtering the weighted restored image.

In penalization block 15, the image parameters values are forced to be distributed within the given interval to produce a sharp, enhanced original image.

The construction of the sharpening filter is then undone in block 16. This is the same procedure as used in block 12 of image enhancement and sharpening method 10, but applied now to the image constructed in the previous block 15 according to formula (46).

Execution of the image enhancement and sharpening method 10 may then return in a recursive loop through constructing sharpening filter block 12, to partially sharpened image block 13, to inverse filtering block 14, and finally to penalization block 15, until the norm of a difference between the observed degraded image  $d$  and numerically predicted degraded image  $d_{pr}$  corresponding to the sharp ideal image  $m_{n+1}$  generated in block 15 reaches a given tolerance value

$$\varepsilon: \|\hat{d} - \hat{d}_{pr}\| = \|\hat{d} - \hat{B}\hat{m}_{n+1}\| = \varepsilon.$$

An important feature of the present invention is that the image deblurring method 10 can be applied to restoration of blurred images of arbitrary origin.

As an example of an application of this method, we present here a result of an image enhancement and sharpening experiment conducted for a seismic image. FIG. 2 shows the ideal seismic image of a geological cross-section. FIG. 3 presents the typical blurred seismic image of the same cross-section which is generated as the result of conventional seismic exploration technique. This image has been processed by the digital image enhancement and sharpening method 10 of the present invention. The reconstructed image is shown in FIG. 4. One can see that this image is practically identical to the original ideal image presented in FIG. 2.

Hence, an important feature of the present invention is the ability to produce a realistic original image from a degraded one. It is believed that the image enhancement and sharpening method 10 of the present invention can be used in optical image processing, for image restoration and sharpening in biomedical, geophysical, astronomical, high definition television, remote sensing, and other industrial applications.

The method of the present invention is based on three new ideas that make it useful in image restoration technique. The first idea is to use a new physical constraint in image restoration procedure based on minimization of the area where strong image parameter variations and discontinuities occur. This constraint provides substantial qualitative improvement in image reconstruction procedures. The second idea is to implement this constraint in a form of weights imposed on image parameters, which makes it practical to

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realize this constraint in a numerical algorithm. The third idea is penalization of the image parameters to keep them within given reasonable limits of variations. Together these three new ideas result in a new method 10 of digital image enhancement and sharpening.

The method 10 of the present invention can be implemented numerically using a general purpose computer programmed to perform the procedures of the blocks 11–16. It can be understood that the method 10 of the present invention may also be performed using dedicated hardware specially designed to solve the constraint minimization problem of the instant method 10.

What is claimed is:

1. In a method of digital image enhancement of a multi-dimensional digital image, said image being represented by a matrix  $[d]$  comprising image parameters, wherein said matrix  $[d]$  is mathematically manipulated to produce a solution to a linear ill-posed inverse problem to reduce blurring, the improvement comprising: imposing a constraint on the solution to the linear ill-posed inverse problem so as to produce an image matrix, said constraint being based upon minimization of the area where strong variations and discontinuities between said image parameters occur.

2. In the method of claim 1, the improvement further comprising: implementing said constraint in the form of weights imposed upon said image parameters.

3. In the method of claim 2, the improvement further comprising: imposing penalization upon said image parameters, thereby to keep said parameters within reasonable limits of variation.

4. In the method of claim 3, the improvement further comprising: solving said ill-posed problem by means of an iterative loop using a programmed computer.

5. In the method of claim 4, stopping said iterative loop when the norm of a difference between the observed degraded image and a numerically predicted degraded image corresponding to an iteratively sharpened image reaches a tolerance value.

6. A method of digital image enhancement of a multidimensional digital image, said image being represented by a matrix  $[d]$  comprising image parameters, comprising the steps of:

- a) initially restoring a digital image  $[m]$  by applying a transposed complex conjugated blurring operator, and an inverse gradient operator to the initial degraded digital image  $[d]$ ;
- b) computing an inverse sharpening filter, thereby to minimize the area where strong image parameter variations and discontinuities occur;
- c) constructing a partially sharpened weighted image by applying said inverse sharpening filter;
- d) constructing an inverse filtered image by inverse filtering said partially sharpened weighted image using said inverse sharpening filter and said inverse gradient operator;
- e) checking the norm of a difference between the observed degraded image and a numerically predicted degraded image corresponding to said sharpened image; if said norm is equal to or less than a user defined tolerance value, then calculating the nonblurred image; otherwise, continuing to step f);
- f) undoing the results of loop steps comprising steps b), c), d), and e); and returning to step b).

7. The method of claim 6 further including, subsequent to step d, and prior to step e), the additional step of imposing penalization to said inverse filtered image, thereby forcing